Leaving Cert Physics: Derivations

1. To Derive: v = u + at

Derivation:

Acceleration = Rate of Change in Velocity

$$Acceleration = \frac{Change\ in\ Velocity}{Time}$$

$$a = \frac{v - u}{t}$$

at = v – u

v = u + at

2. To Derive: s = ut + ½ at^2

Derivation:

$$Average\ Velocity = \frac{Initial\ Velocity + Final\ Velocity}{2}$$

$$V_{average} = \frac{u + v}{2}$$

But v = u + at

$$V_{average} = \frac{u + u + at}{2}$$

$$V_{average} = \frac{Displacement\ (s)}{Time\ (t)}$$

$$(V_{average})(t) = Displacement\ (s)$$

$$\left(\frac{u + u + at}{2}\right)(t) = s$$

$$\left(\frac{2u + at}{2}\right)(t) = s$$

$$\left(\frac{2ut + at^2}{2}\right) = s$$

ut + ½ at^2 = s

3. To Derive: $v^2 = u^2 + 2as$

Derivation:

We know: $v = u + at$ (Square both sides)

$v^2 = u^2 + 2uat + a^2t^2$ (Factorise out 2a)

$v^2 = u^2 + 2a(ut + ½ at^2)$ (We know $s = ut + ½ at^2$)

$v^2 = u^2 + 2as$

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4. To Derive: F = ma is a special case of Newton's Second Law
[Use Newton's second law to establish the relationship: force = (mass)(acceleration)]

Derivation:

From Newton's 2nd law: Force is proportional to the rate of change of momentum

Force $\propto$ rate of change of momentum

$$Force \propto \left(\frac{Change\ of\ momentum}{Time}\right)$$

$$F \propto \left(\frac{mv - mu}{t}\right)$$

$$F \propto m\left(\frac{v - u}{t}\right)$$

$F \propto ma$

$F = k\,(ma)$

$F = ma$

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5. To Derive: $v = r\,\omega$

Derivation:

We define θ (in radians) as (length of the arc)/radius: $\theta = \frac{S}{r}$

Divide both sides by t : $\frac{\theta}{t} = \frac{S}{tr}$

But $\omega = \frac{\theta}{t}$ and $v = \frac{s}{t}$: $\omega = \frac{v}{r}$

Cross-multiply: $v = r\,\omega$

6. To Derive: Relationship between Periodic Time (T) and Radius (R) for a satellite in orbit

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

Derivation: We compare two formulae which we have for Force:

The first is the *Universal Gravitational Force* formula: $F_g = \frac{Gm_1 m_2}{r^2}$

The second is the *Centripetal Force* formula: $F_c = \frac{mv^2}{r}$

Equate both forces (because both equations apply to satellite motion)

$$\frac{Gm_1 m_2}{r^2} = \frac{mv^2}{r}$$

Cancel one of the "m"s and an "r" from each side:

$$\frac{Gm}{r} = v^2$$

Meanwhile, $Velocity = \frac{Distance}{Time}$

In this case, the distance is the circumference of the orbit (2πr) and the time is T

$$v = \frac{2\pi r}{T} \qquad \Rightarrow \qquad v^2 = \frac{4\pi^2 r^2}{T^2}$$

Let $v^2 = v^2$:

$$\frac{Gm}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{Gm}$$

7. To Derive: To show that any object that obeys Hooke's Law will also execute SHM

Derivation: Start with the equation for Hooke's Law: $F \propto -s$ $\Rightarrow$ $F = -k\,s$

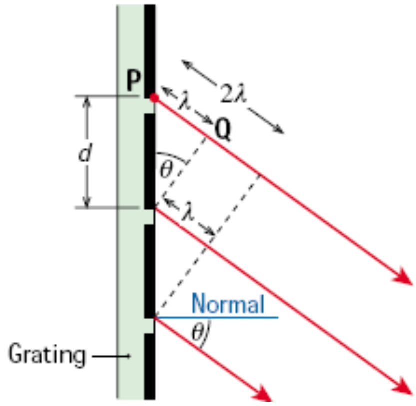
But F = ma , so therefore ma = -k s

Now divide both sides by m : $a = -\frac{k}{m} s$

This is equivalent to the equation for S.H.M. where the constant ω^2 in this case is k/m.

8. **To Derive:** Diffraction grating formula: $n\lambda = d \sin\theta$

Derivation:



From the diagram we can see that:

For constructive interference to occur, the extra path length that the top ray travels must be an integer number of wavelengths (**$n\lambda$**) {Eqn (1)}

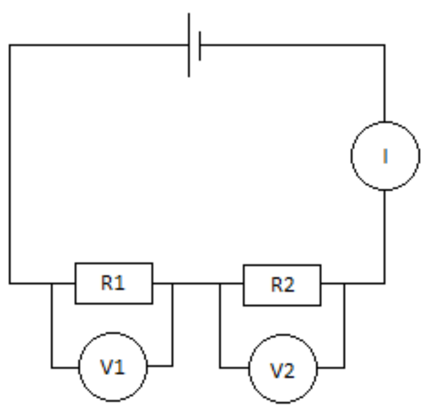
Using trigonometry, this extra path length is equal to **$d \sin\theta$,** where d is the slit width {Eqn (2)}

Equating (1) and (2) gives us **$n\lambda = d \sin\theta$**

9. **To Derive:** Derive an expression for the effective resistance of the two resistors connected in series.

$$R_{Total} = R_1 + R_2$$

Derivation:



Apply Ohm's Law to each resistor:

$V_1 = I\,(R_1)$ and $V_2 = I\,(R_2)$

Since the voltages are in series: $V_{total} = V_1 + V_2$

$$V_{total} = I\,(R_1) + I\,(R_2)$$

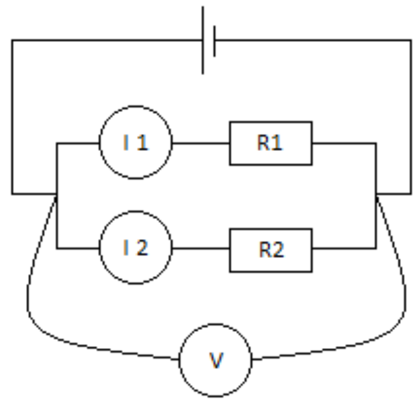
$I\,(R_{total}) = I\,(R_1 + R_2)$ (Divide across by I)

So, $R_{Total} = R_1 + R_2$

10. To Derive: Derive an expression for the effective resistance of the two resistors connected in parallel.

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Derivation:



Apply Ohm's Law to each resistor:

$V = I_1 (R_1) \rightarrow \frac{V}{R_1} = I_1$

$V = I_2 (R_2) \rightarrow \frac{V}{R_2} = I_2$

$I_{total} = I_1 + I_2$

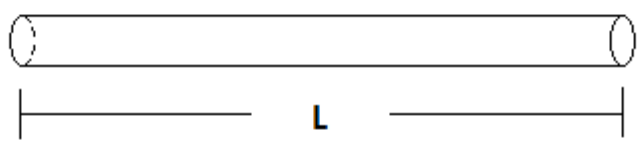
$$\frac{V}{R_{Total}} = \frac{V}{R_1} + \frac{V}{R_2}$$

(Divide across by V)

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

11. To Derive: Force on a moving charge: F = Bqv

Derivation:



Consider a section of conductor of length *l* through which a current *I* is flowing.

If *q* is the charge which carries the current in this section of the conductor, then:

$I = q / t$, (remember q = It, where t is the time it takes the charge *q* to travel a distance *l*).

The average velocity with which the charge flows is given by $v = l / t$, i.e. $l = vt$.

Substituting into the primary equation which we have for force (F = BIL), we get

$$F = B \times q/t \times vt$$

i.e. $F = Bqv$