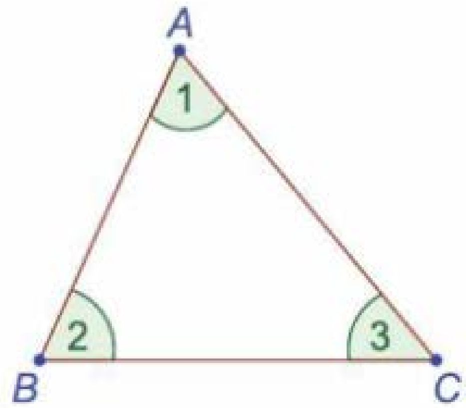
Theorem 4

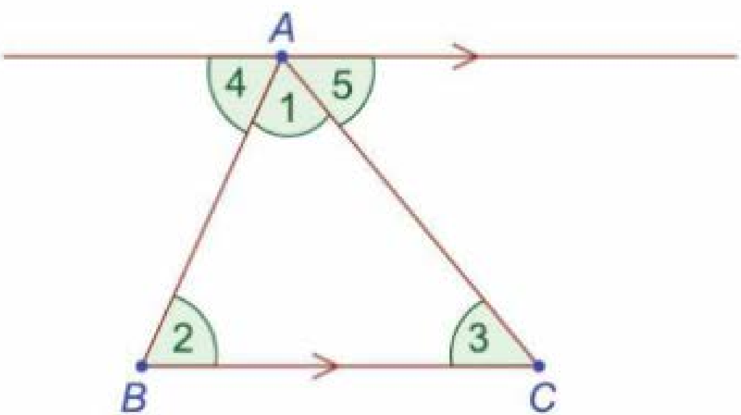
The angles in any triangle add up to 180°.



Given: A triangle with angles ∠1, ∠2 and ∠3.

To prove: $|\angle 1| + |\angle 2| + |\angle 3| = 180°$.

Construction: Draw a line through *A*, parallel to *BC*. Label angles 4 and 5.



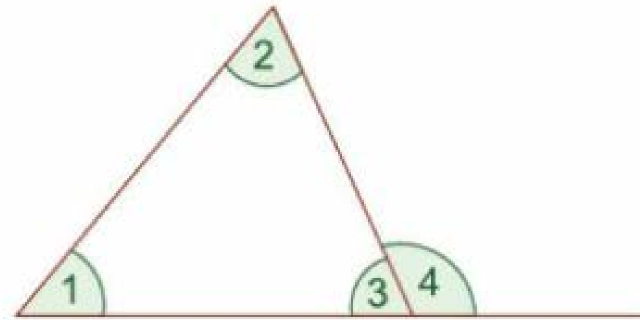
Proof:

Statement	Reason
$\|\angle 4\| + \|\angle 1\| + \|\angle 5\| = 180°$	Straight angle
$\|\angle 2\| = \|\angle 4\|$	Alternate
$\|\angle 3\| = \|\angle 5\|$	Alternate
$\Rightarrow \|\angle 4\| + \|\angle 1\| + \|\angle 5\| = \|\angle 2\| + \|\angle 1\| + \|\angle 3\|$	
$\Rightarrow \|\angle 1\| + \|\angle 2\| + \|\angle 3\| = 180°$	
Q.E.D.	

Theorem 6

Each exterior angle of a triangle is equal to the sum of the interior remote angles.

Given: A triangle with interior angles $\angle 1$, $\angle 2$ and $\angle 3$, and an exterior angle $\angle 4$.



To prove: $|\angle 1| + |\angle 2| = |\angle 4|$.

Proof:

Statement	Reason
$\lvert\angle 3\rvert + \lvert\angle 4\rvert = 180°$	Straight angle
$\lvert\angle 1\rvert + \lvert\angle 2\rvert + \lvert\angle 3\rvert = 180°$	Angles in a triangle
$\Rightarrow \lvert\angle 1\rvert + \lvert\angle 2\rvert + \lvert\angle 3\rvert = \lvert\angle 3\rvert + \lvert\angle 4\rvert$	Both = 180°
$\Rightarrow \lvert\angle 1\rvert + \lvert\angle 2\rvert = \lvert\angle 4\rvert$	Subtracting $\lvert\angle 3\rvert$
Q.E.D.	

Theorem 9

In a parallelogram, opposite sides are equal and opposite angles are equal.

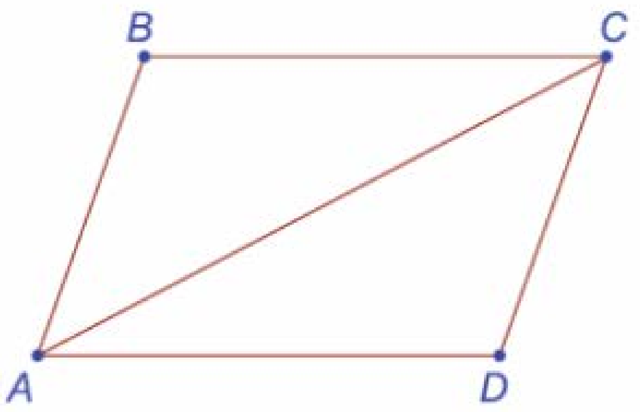
Given: A parallelogram $ABCD$.

To prove:

(i) $|AB| = |CD|$ and $|BC| = |AD|$ (opposite sides are equal)

(ii) $|\angle ABC| = |\angle ADC|$, $|\angle BAD| = |\angle BCD|$ (opposite angles are equal)

Construction: Draw the diagonal $[AC]$.

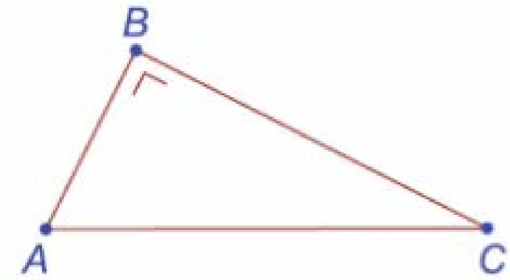


Proof:

Statement	Reason
$\|\angle BCA\| = \|\angle CAD\|$	Alternate
$\|AC\| = \|AC\|$	Common (shared)
$\|\angle BAC\| = \|\angle ACD\|$	Alternate
$\Rightarrow \Delta BAC \equiv \Delta ADC$	ASA
$\Rightarrow \|AB\| = \|CD\|$ and $\|BC\| = \|AD\|$	Corresponding sides
Also, $\|\angle ABC\| = \|\angle ADC\|$	Corresponding angle
Similarly, $\|\angle BAD\| = \|\angle BCD\|$	
Q.E.D.	

Theorem 14: Theorem of Pythagoras

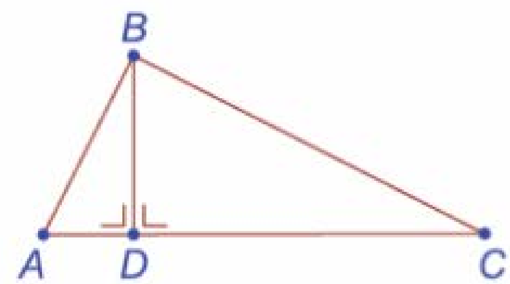
In a right-angled triangle, the square of the hypotenuse is the sum of the squares of the other two sides.



Given: A right-angled triangle ABC with $|\angle ABC| = 90°$.

To prove: $|AC|^2 = |AB|^2 + |BC|^2$.

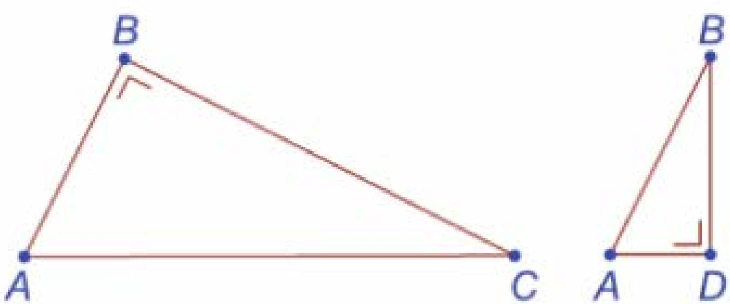
Construction: Draw $BD \perp AC$.



Proof:

Step 1

Consider the triangles ABC and ADB.



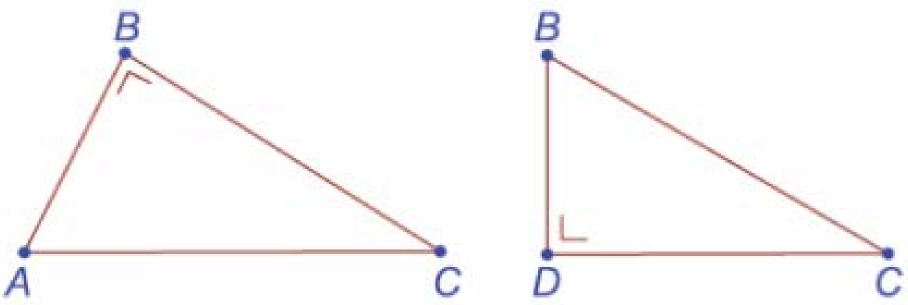
$\lvert\angle ABC\rvert = \lvert\angle ADB\rvert$	90°
$\lvert\angle BAC\rvert = \lvert\angle BAD\rvert$	Common

$\therefore$ ΔABC and ΔADB are similar.

Statement	Reason
ΔABC and ΔADB are similar.	Construction
$\Rightarrow \frac{\lvert AC\rvert}{\lvert AB\rvert} = \frac{\lvert AB\rvert}{\lvert AD\rvert}$	Theorem
$\Rightarrow \lvert AB\rvert . \lvert AB\rvert = \lvert AC\rvert . \lvert AD\rvert$	
$\Rightarrow \lvert AB\rvert^2 = \lvert AC\rvert . \lvert AD\rvert$	

Step 2

Consider the triangles *ABC* and *BDC*.



$\lvert\angle ABC\rvert = \lvert\angle BDC\rvert$	90°
$\lvert\angle ACB\rvert = \lvert\angle DCB\rvert$	Common

$\therefore$ ΔABC and ΔBDC are similar.

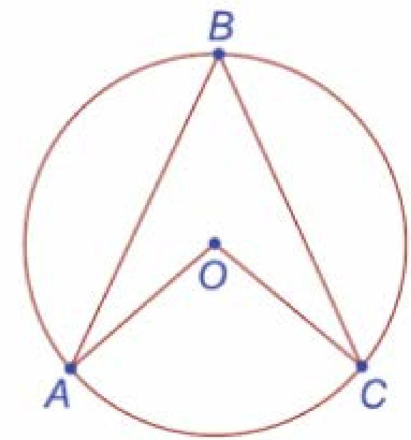
Statement	Reason
ΔABC and ΔBDC are similar.	Construction
$\Rightarrow \dfrac{\lvert AC\rvert}{\lvert BC\rvert} = \dfrac{\lvert BC\rvert}{\lvert DC\rvert}$	Theorem
$\Rightarrow \lvert BC\rvert . \lvert BC\rvert = \lvert AC\rvert . \lvert DC\rvert$	
$\Rightarrow \lvert BC\rvert^2 = \lvert AC\rvert . \lvert DC\rvert$	

Step 3

$\lvert AB\rvert^2 + \lvert BC\rvert^2 = \lvert AC\rvert . \lvert AD\rvert + \lvert AC\rvert . \lvert DC\rvert$
$= \lvert AC\rvert . (\lvert AD\rvert + \lvert DC\rvert)$
$\Rightarrow \lvert AB\rvert^2 + \lvert BC\rvert^2 = \lvert AC\rvert . \lvert AC\rvert$ (Since $\lvert AD\rvert + \lvert DC\rvert = \lvert AC\rvert$)
$\lvert AB\rvert^2 + \lvert BC\rvert^2 = \lvert AC\rvert^2$
Q.E.D.

Theorem 19

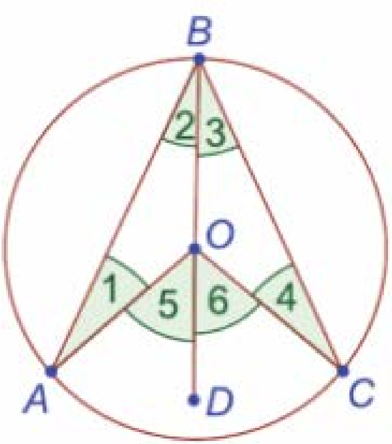
The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.



Given: A circle with centre O and an arc AC. A point B on the circle.

To prove: $|\angle AOC| = 2|\angle ABC|$.

Construction: Join B to O and continue to a point D. Label angles 1, 2, 3, 4, 5 and 6.



$|\angle AOC| = |\angle 5| + |\angle 6|$

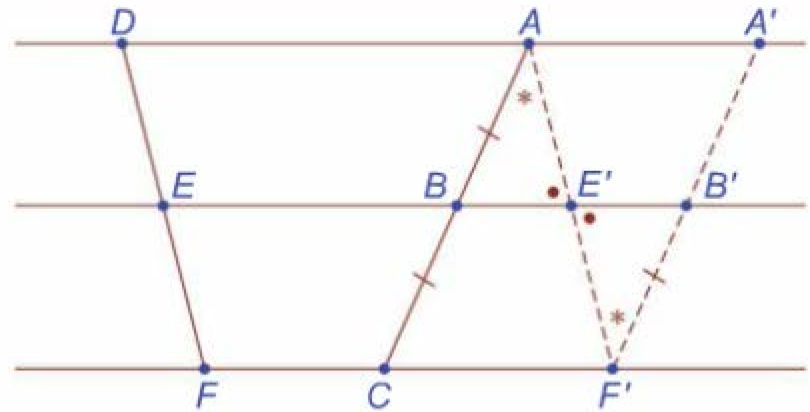
$|\angle ABC| = |\angle 2| + |\angle 3|$

Proof:

Statement	Reason
$\lvert OA\rvert = \lvert OB\rvert$	Radii
$\lvert\angle 1\rvert = \lvert\angle 2\rvert$	Isosceles triangle
$\lvert\angle 5\rvert = \lvert\angle 1\rvert + \lvert\angle 2\rvert$	Exterior angle
$\Rightarrow \lvert\angle 5\rvert = 2\lvert\angle 2\rvert$	Since $\lvert\angle 1\rvert = \lvert\angle 2\rvert$
Similarly, $\lvert\angle 6\rvert = 2\lvert\angle 3\rvert$	
$\lvert\angle 5\rvert + \lvert\angle 6\rvert = 2\lvert\angle 2\rvert + 2\lvert\angle 3\rvert$	
$\Rightarrow \lvert\angle 5\rvert + \lvert\angle 6\rvert = 2(\lvert\angle 2\rvert + \lvert\angle 3\rvert)$	
$\lvert\angle AOC\rvert = 2\lvert\angle ABC\rvert$	
Q.E.D.	

Theorem 11

If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.



Given: $AD \parallel BE \parallel CF$, as in the diagram, with $|AB| = |BC|$.

To prove: $|DE| = |EF|$.

Construction: Draw $AE' \parallel DE$, cutting EB at E' and CF at F'.

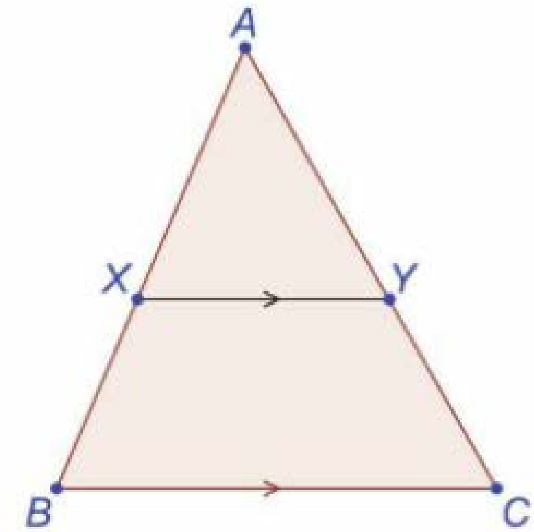
Draw $F'B' \parallel AB$, cutting EB at B', as in the diagram.

Proof:

Statement	Reason
$\lvert B'F'\rvert = \lvert BC\rvert$	Opposite sides in a parallelogram
$= \lvert AB\rvert$	By assumption
$\lvert \angle BAE'\rvert = \lvert \angle E'F'B'\rvert$	Alternate angles
$\lvert \angle AE'B\rvert = \lvert \angle F'E'B'\rvert$	Vertically opposite angles
$\therefore$ $\Delta ABE'$ is congruent to $\Delta F'B'E'$	ASA
Therefore, $\lvert AE'\rvert = \lvert F'E'\rvert$.	
But $\lvert AE'\rvert = \lvert DE\rvert$ and $\lvert F'E'\rvert = \lvert FE\rvert$	Opposite sides in a parallelogram
$\therefore$ $\lvert DE\rvert = \lvert EF\rvert$	
Q.E.D.	

Theorem 12

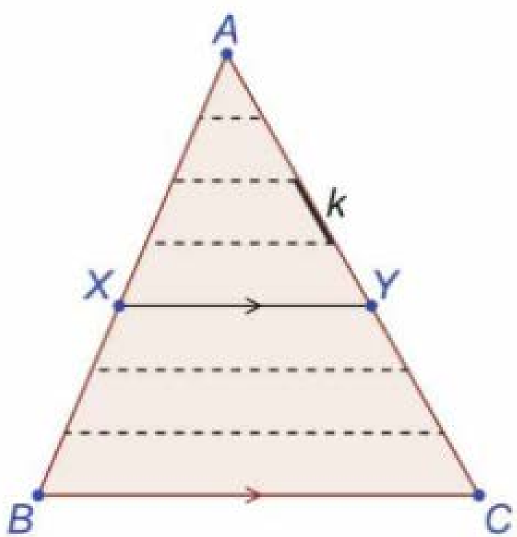
Let ABC be a triangle. If a line l is parallel to BC and cuts $[AB]$ in the ratio $s : t$, then it also cuts $[AC]$ in the same ratio.



Given: A triangle ABC and a line XY parallel to BC which cuts $[AB]$ in the ratio $s : t$.

To prove: $|AY| : |YC| = s : t$

Construction: Divide $[AX]$ into s equal parts and $[XB]$ into t equal parts. Through each point of division, draw a line parallel to BC.



Proof: According to Theorem 11, the parallel lines cut off segments of equal length along $[AC]$.

Let k be the length of each of these equal segments.

$\Rightarrow |AY| = sk$ and $|YC| = tk$

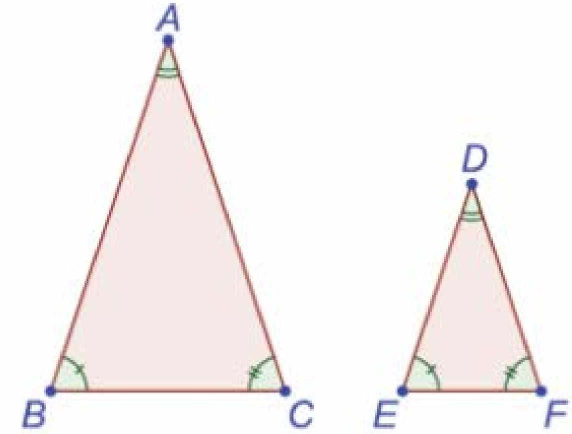
$\Rightarrow |AY| : |YC| = sk : tk = s : t$

Q.E.D.

Theorem 13

If two triangles *ABC* and *DEF* are similar, then their sides are proportional, in order:

$$\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$$

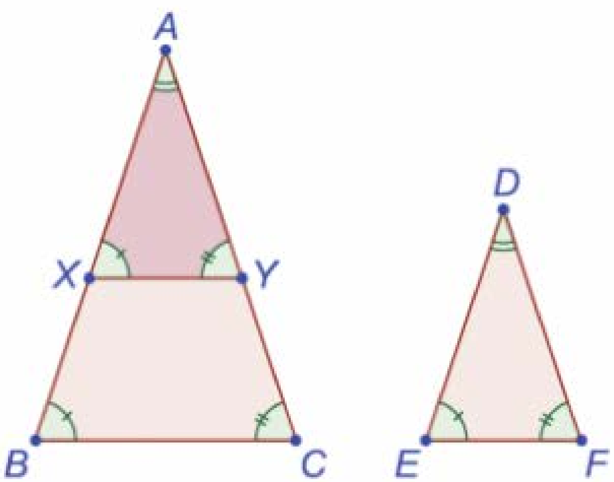


Given: Similar triangles *ABC* and *DEF*.

To prove: $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$

Construction: Assume triangle *DEF* is smaller than triangle *ABC*.

- Mark a point *X* on [*AB*] such that $|AX| = |DE|$, and mark a point *Y* on [*AC*] such that $|AY| = |DF|$ as shown.
- Draw [*XY*].



Proof:

Statement	Reason
ΔAXY is congruent to ΔDEF.	SAS
$\Rightarrow \lvert\angle AXY\rvert = \lvert\angle ABC\rvert$	
$\Rightarrow XY \parallel BC$	Corresponding angles equal
$\Rightarrow \frac{\lvert AB\rvert}{\lvert AX\rvert} = \frac{\lvert AC\rvert}{\lvert AY\rvert}$	Theorem 12
But $\lvert AX\rvert = \lvert DE\rvert$ and $\lvert AY\rvert = \lvert DF\rvert$.	Construction
$\Rightarrow \frac{\lvert AB\rvert}{\lvert DE\rvert} = \frac{\lvert AC\rvert}{\lvert DF\rvert}$	
Similarly, $\frac{\lvert BC\rvert}{\lvert EF\rvert} = \frac{\lvert AB\rvert}{\lvert DE\rvert}$	
$\Rightarrow \frac{\lvert AB\rvert}{\lvert DE\rvert} = \frac{\lvert BC\rvert}{\lvert EF\rvert} = \frac{\lvert AC\rvert}{\lvert DF\rvert}$	
Q.E.D.	