Algebra 1 Cheat Sheet

Polynomial

An expression of the form:

$$a_x x^n + a_{n-1}x^{n-1} + \cdots + a^2x^2 + a_1x + a_0$$

Where $a_i \in \mathbb{R}$; $i = 0, 1, 2, \ldots, n$; $n \in \mathbb{N}$.

Types Of Polynomials

Linear *(degree 1)*	$ax + b$
Quadratic *(degree 2)*	$ax^2 + bx + c$
Cubic *(degree 3)*	$ax^3 + bx^2 + cx + d$

Factoring

Difference Of Two Squares	$a^2 - b^2 = (a + b)(a - b)$
Difference Of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Sum Of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Polynomial Long Division

$(x^3 + x^2 - 2x) \div (x - 1)$

Solution:

$$\begin{array}{r|l} & x^2 + 2x \\ \hline x - 1 & x^3 + x^2 - 2x \\ & -x^3 + x^2 \\ \hline & 2x^2 - 2x \\ & -2x^2 + 2x \\ \hline & 0 \end{array}$$

Algebraic Fractions

$$a\left(\frac{b}{c}\right) = \frac{ab}{c} \qquad \frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b} \qquad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \qquad \frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab + ac}{a} = b + c, a \neq 0$$

Binomial Coefficient

The binomial coefficient represents the number of ways you can choose r objects from a group of n objects.

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \qquad \binom{n}{r} = \binom{n}{n-r}$$

$$\binom{n}{r} = \frac{n(n-1)(n-2)\ldots(n-(k-1))}{k(k-1)(k-2)\ldots 1}$$

Binomial Theorem

For any positive integer n,

$$\begin{aligned}(a+b)^n &= \sum_{m=0}^{n} \binom{n}{m} a^{n-m}b^n \\ &= \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \cdots + \binom{n}{n}b^n\end{aligned}$$

Algebraic Identities

In an **identity**, all coefficients of like powers are equal.

An identity must be true for *all values* of the independent variables

Example: $3x + 7 = ax + b$ implies $a = 3$ and $b = 7$.

Manipulating Formulae

What you do to one side of an equation, you must do to the other.

In terms of x	Write with x as the independent variable
The subject of	Make it on it's own on one side of the equation
As a function of	same as 'in terms of'

Solving Linear Equations

Linear equations can be solved by making x with subject of the equation.

Example: Solve $4x + 2 = 14$

$$\begin{aligned} & 4x + 2 = 14 \\ \implies & 4x = 14 - 2 = 12 \\ \implies & x = \frac{12}{4} = 3 \end{aligned}$$

Systems of Equations With Three Variables

1. Reduce the three equations to two by eliminating one of the unknowns.
2. Choose an unknown to isolate.
3. Eliminate this unknown from all three equations, taking them two at a time.
4. Solve these two equations.
5. Use the solutions as values in the original equations
6. Check your solution!