Uniform Acceleration - Leaving Cert Applied Maths

Adam Kelly

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In this section, **linear motion** with **constant acceleration** is studied. *Motion* is a change in the position of a particle. *Linear* motion is motion in one dimension. In the questions these apply to, this means up and down, forwards and backwards and so on.

Summary

- v: final velocity
- u: initial velocity
- a: acceleration
- s: displacement
- t: time

These quantities are related through the following equations:

$$v = u + at \tag{1}$$
$$s = \frac{(u+v)}{2}t \tag{2}$$
$$s = ut + \frac{1}{2}at^2 \tag{3}$$
$$v^2 = u^2 + 2as \tag{4}$$
$$\tag{5}$$

1 Vector Quantities and Scalar Quantities

In the leaving cert applied maths course, there are two types of quantities to think about, vector quantities and scalar quantities.

DEFINITION **Scalar Quantity.**

A quantity with magnitude only.

DEFINITION **Vector Quantity.**

A quantity with both magnitude and direction

Examples of each of these are

Scalar Quantity	Vector Quantity
Distance	Displacement

Scalar Quantity	Vector Quantity
Speed	Velocity
Length	Acceleration
Area	Force

You should note that sometimes a vector quantity a may be referred to as $\vec{a}$.

2 Displacement, Velocity and Acceleration

DEFINITION

Displacement.

The change in the position of an object,

$$s = \Delta x = x_f - x_i$$

Where x_f refers to the value of the final position, x_i refers to the initial position, and both s and Δx refer to displacement. Displacement is a vector quantity. In the course, displacement is usually measured in metres (m).

DEFINITION

Average Velocity.

$$\overline{v} = \frac{s}{t} = \frac{\textit{change in position}}{\textit{change in time}} = \frac{\Delta x}{\Delta t}$$

Velocity is a vector quantity. It's unit is usually ms^{-1}.
You should know the difference between average velocity and instantaneous velocity. Instantaneous velocity is just the velocity at a specific instant in time, in other words, the average velocity over an infinitesimally small time interval.

DEFINITION

Instantaneous Velocity.

Velocity at a specific instant in time.

$$v = \lim_{\Delta t \to 0} \frac{s}{\Delta t}$$

DEFINITION

Acceleration.

Accleration is the rate of change of velocity.

$$a = \frac{\Delta v}{\Delta t} = \frac{v - u}{t}$$

Here, v represents the final velocity and u represents the initial velocity. This convention of u and v are used throughout the course. The unit for acceleration is usually ms^{-2} (metres per second per second).

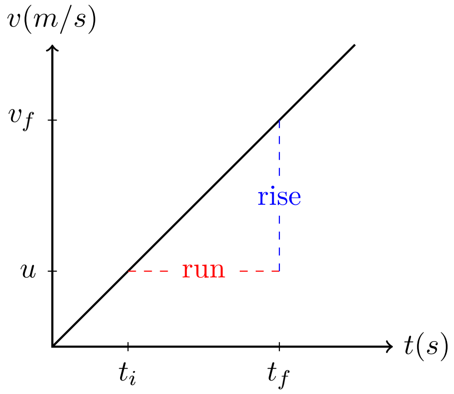
3 Velocity-Time Graphs

Velocity-Time graphs, while hard to interpret, are quite useful. They have a number of properties:

- The slope of a velocity-time graphs represents acceleration
- The area under a velocity-time graph represents the displacement of the object

3.1 Slope Of A Velocity-Time Graph

Consider the following velocity-time graph:

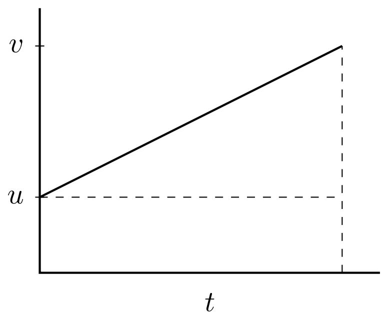


Since $\frac{\Delta v}{\Delta t}$ is the definition of acceleration, the slope of a velocity graph must equal the acceleration of the object.

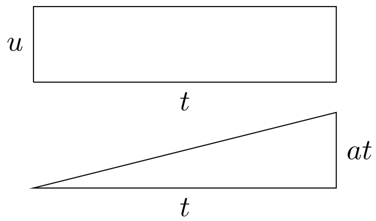
3.2 Area Under A Velocity-Time Graph

THEOREM 3.1 **Area Under A Velocity-Time Graph.**

The area under a velocity-time graph is equal to the distance travelled.



Proof. Since $v = u + at$ (see equation 1), the gap between u and v is of length at. The area under the curve is the sum of the area of a rectangle and a triangle.



$$\text{area of rectangle} = ut$$

$$\text{area of triangle} = \frac{a}{2}t^2$$

Adding these,

$$\text{total area} = ut + \frac{a}{2}t^2$$

Which is s, the distance covered (see equation 3). QED

4 Kinematic Formulae

The kinematic formulae are a set of equations that relate the five kinematic variables, s: displacement, t: time interval, u: initial velocity, v: final velocity and a: constant acceleration.
The formulae (and their proofs) are stated here.

THEOREM 4.1 **First Kinematic Formula.**

$$v = u + at$$

Proof. This formula can be derived by starting with the definition of acceleration, and solving for v.

$$\begin{aligned} a &= \frac{v-u}{t} \\ \implies v - u &= at \\ \implies v &= at + u \end{aligned}$$

QED

THEOREM 4.2 **Second Kinematic Formula.**

$$s = (\frac{u+v}{2})t$$

Proof. Since the acceleration is constant, the average velocity can be found by averaging the initial and final velocities.

$$\overline{v} = \frac{u+v}{2}$$

by the definition of v,

$$v = \frac{s}{t} \implies s = vt = (\frac{u+v}{2})t$$

QED

THEOREM 4.3 **Third Kinematic Formula.**

$$s = ut + \frac{a}{2}t^2$$

Proof. This proof can be done in a number of ways. For the geometric proof, see theorem 3.2. I prefer a proof involving a small amount of simple calculus. By integrating the first kinematic equation (for velocity), you will get the displacement:

$$\begin{aligned}\int v dt &= \int u + at dt \\ &= ut + \frac{a}{2}t^2\end{aligned}$$

QED

THEOREM 4.4 **Fourth Kinematic Formula.**

$$v^2 = u^2 + 2as$$

Proof. This can be proven by solving the first kinematic formula for t.

$$v = u + at \implies t = \frac{v-u}{a}$$

Taking this and using it again with the first formula

$$\begin{aligned}v^2 &= (u+at)^2 \\ &= u^2 + 2uat + a^2t^2 \\ &= u^2 + 2a(ut + \frac{a}{2}t^2) \\ &= u^2 = 2as\end{aligned}$$

QED