Coordinate Geometry: The Line

Distance Between Points

Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$\pm\frac{-b+\sqrt{b^2-4ac}}{2a}$$

Midpoints of a Line Segment

Midpoint of line segment between $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$$

Slope of a Line

Choose two points for (x_1, y_1) and (x_2, y_2). The slope is:

$$m = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{y_2-y_1}{x_2-x_1}$$

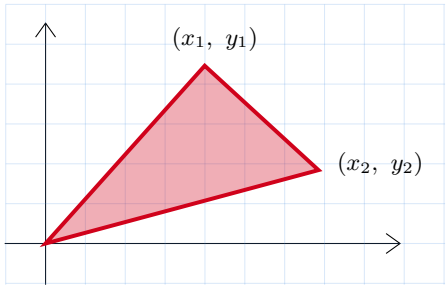
Parallel and Perpendicular Slopes

1. Two lines are $\parallel$ (parallel) if their slopes are equal.
2. Two lines are $\perp$ (perpendicular) if their slopes are negative reciprocals ($m_2 = -1/m_1$).

Area of a Triangle

The area of a triangle with vertices $(0,0)$, (x_1, y_1), and (x_2, y_2) is:

$$\text{Area} = \frac{1}{2}|x_1y_2 - x_2y_1|$$

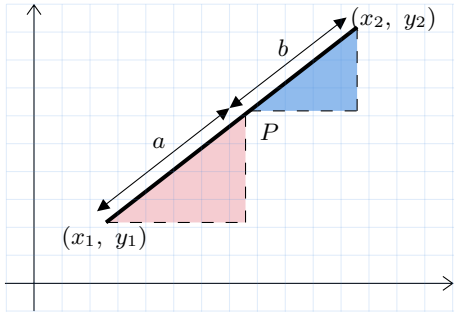


You may need to translate the triangle to get one of the vertices to $(0,0)$.

Dividing a Line Segment by a Given Ratio

Below, the point P divides the segment in the ratio $a : b$. The coordinates of P are:

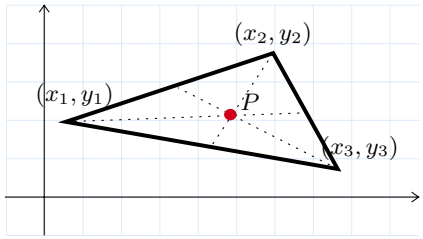
$$P = (\frac{bx_1+ax_2}{b+a}, \frac{by_1+ay_2}{b+a})$$



Triangle Concurrency: Centroid

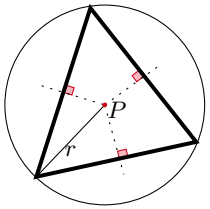
Intersection of the medians. Point given by

$$P = (\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3})$$



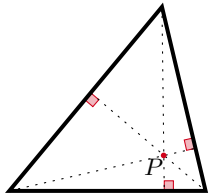
Triangle Concurrency: Circumcentre

Intersection of the perpendicular bisectors, and center of the circumcircle.



Triangle Concurrency: Orthocentre

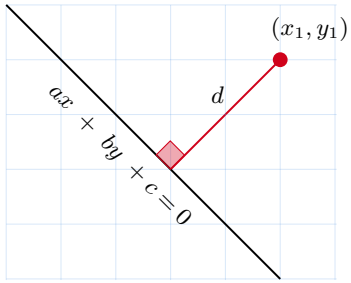
The intersection of the perpendiculars from the vertices to the opposite sides.



Perpendicular Distance: Point to a Line

The perpendicular distance d from a point (x_1, y_1) to a line $ax + by + c = 0$ is:

$$d = \frac{|ax_1+by_2+c|}{\sqrt{a^2+b^2}}$$



Angle Between Two Lines

The angle θ between two lines with slopes m_1 and m_2 as shown is given by:

$$\tan\theta = \pm\frac{m_1-m_2}{1+m_1m_2}$$